Radiophysics and Quantum Electronics, Vol. 38, Nos. 1–2, 1995

PERIODIC EVOLUTION OF SPACE CHAOS IN THE ONE-DIMENSIONAL COMPLEX GINZBURG–LANDAU EQUATION

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Periodic evolution of the space chaos in a one-dimensional distributed system represented by the complex Ginzburg–Landau equation is studied. There exists a region of parameters where spatially chaotic distribution of the field varies periodically with time, and the boundaries of this region are determined. The regime of periodic space chaos was found to exist only for certain initial conditions. A system of ordinary differential equations that describes the space chaos is derived.

1. INTRODUCTION

Among the most widely accepted models in the theory of nonequilibrium media is the complex Ginzburg–Landau equation, which describes the behavior of a distributed system in the vicinity of an Andronov–Hopf bifurcation point:

$$\partial_t a = a - (1 + i\beta)|a|^2 a + (1 + i\alpha)\partial^2_{xx} a. \tag{1}$$

Equation (1) depends on two real parameters, α and β, and, even in the 1-D case, it describes a great number of phenomena, for example, regimes of phase and amplitude turbulence, the hysteresis phenomenon, and the space-time intermittence (see, for example, [2, 3]. In this paper, we study another interesting phenomenon — the space chaos periodically oscillating in time (see also [1]).

2. TIME SYNCHRONIZATION OF IRREGULAR PATTERNS

Let us consider Eq. (1) in the region $G_L = \{-L \le x \le L\}$ with periodic boundary conditions at the boundary $x = \pm L$. As the initial space, we take $\mathcal{L}_4(G_L)$ — a set of complex functions which are continuous in G_L with the norm

$$\| a \| = \Big(\int_{G_L} |a|^4 dx \Big)^{1/4}. \tag{2}$$

Equation (1) was integrated over the region G_L with periodic boundary conditions using the pseudo-spectral method [4].

Figure 1 shows the results of a computer-aided experiment. The symbol "o" denotes the points at which a regime of the space chaos with the complex amplitude $a(x,t)$ periodically oscillating in time was established, and the symbol "*" denotes the points at which the space-time chaos was established (the Benjamin–Feir line, the boundary of the amplitude and phase turbulence regions, and the boundary of the hysteresis region are also indicated [3]). The above points do not exhaust the entire set of parameter values, for which the regime in question can be realized. This relates to the fact that the periodic regime is not set up for all initial values, and that the time for the periodic regime to set up is substantially great ($T \sim 10^4$) and increases as the distance to the upper boundary of the periodicity region decreases. The lower boundary seems to match the existence boundary of the amplitude turbulence regime [3] (dashed line in Fig. 1).

Institute of Applied Physics, Russian Academy of Sciences, Nizhny Novgorod. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 38, Nos. 1–2, pp. 37-43, January–February, 1995. Original article submitted January 12, 1995.

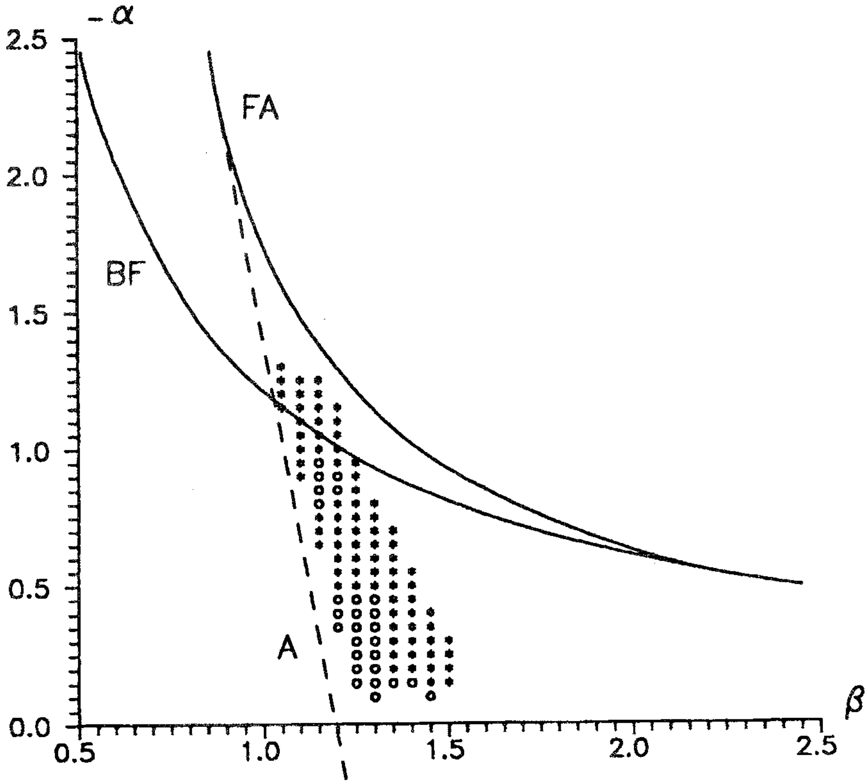

Fig. 1.

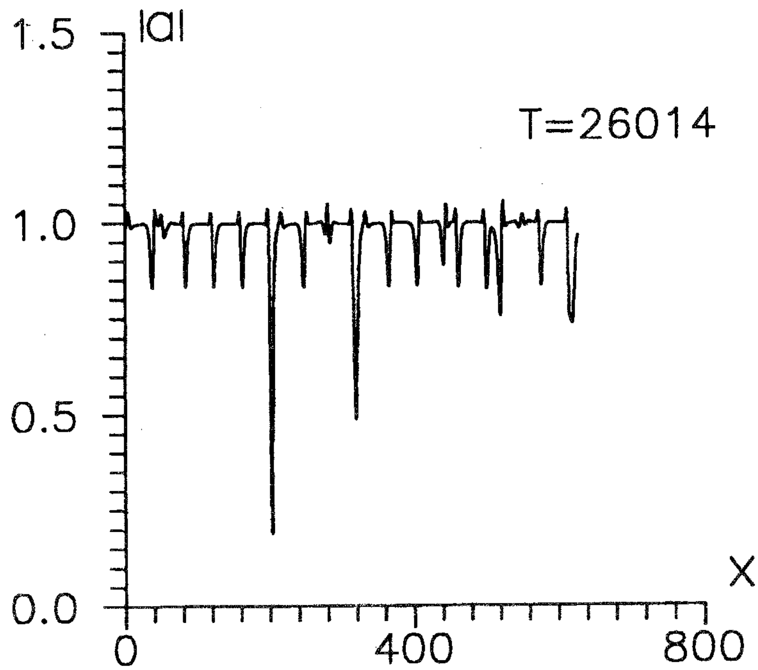

Fig. 2.

The periodicity region shown in Fig. 1 was constructed for a system with length $2L = 600$. A system with length $2L = 6000$ was also considered. The oscillation period was invariant with respect to the system length for all cases.

Figure 2 shows an instantaneous shot of the field amplitude under the established regime for parameter values from the periodicity region ($\beta = 1.5,\ \alpha = -0.9$). The dependence of the real part of the field a on

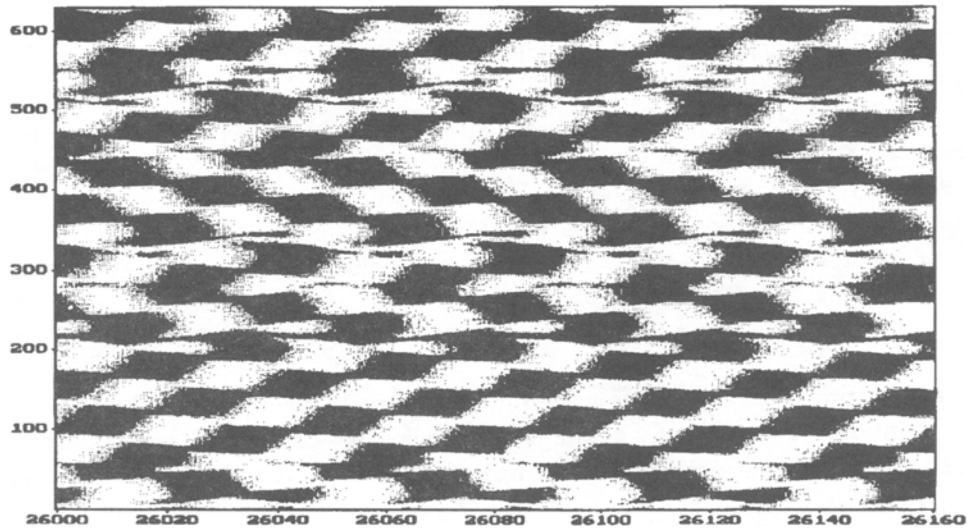

Fig. 3.

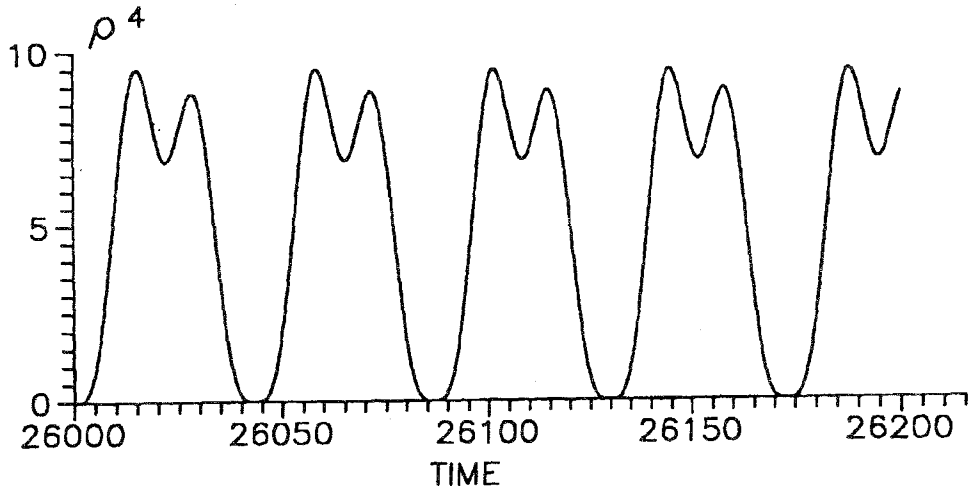

Fig. 4.

the coordinate and time on the $(x - t)$-plane for the same α and β is given in Fig. 3, where a developed amplitude turbulence is evident. The time evolution of the space distribution (Fig. 2) is complex, involves the birth and the disappearance of structures, but is regular in time.

To analyze the above oscillations, we introduce the distance in the space $\mathcal{L}_4(G_L)$. Using the norm (2), we have

$$\rho(u,v) = \Big(\int_{G_L} |u - v|^4 dx\Big)^{1/4}. \tag{3}$$

Figure 4 shows the dependence of $\rho^4\big(a(x,t_0),\ a(x,t_0+t)\big)$ on time t for $\beta = 1.5$ and $\alpha = -0.9$. As one would expect, this dependence is of the oscillatory character, the distance between the functions $a(x,t_0)$ and $a(x,t_0+t)$ vanishes periodically, and the oscillation period ($T \approx 43$) coincides with the variation period of the variable $a(x,t)$ (see Fig. 3).

It was mentioned that the regime of a space chaos periodically oscillating in time is not established for all the initial conditions. However, when this was the case, the oscillation period was the same, although the particular form of the space distribution differed for various initial conditions. The dependence of the distance between the functions $a(x,t_0)$ and $a'(x,t_0+t)$ satisfying various initial conditions is periodic (with the same period), but $\rho\Big(a(x,t_0),\, a'(x,t_0+t)\Big)$ does not vanish.

Due attention should be paid to the structural stability of our regime, i. e., the stability relative to the small perturbations on the right side of Eq. (1). We investigated the stability of the periodic regime relative to the perturbations on the right side of Eq. (1) in the form $\pm\delta|a^4|a$ with $\delta = 0.005$. The regime of the time-periodic chaos was found to be structurally stable. The period of oscillations is a continuous function of the right side of Eq. (1), although the form of oscillations changes markedly with the perturbation sign.

3. DISCUSSION

The described regime can appear as a result of the Andronov–Hopf bifurcation of the stable space distribution, which incorporates a family of the Nozaki–Bekki darkening solitons [5]. This scenario is quite possible if we consider Eq. (1) as a chain of related oscillators, and the regime studied can be interpreted as a synchronization of oscillators.

The change to chaos in the system is performed through intermittence. In the vicinity of the boundary of the periodicity region, a regime is observed under which the spatial location of the aperiodicity islands changes with time, and, therefore, periodic oscillations are interrupted by chaotic bursts.

The space chaos oscillating periodically in time found in the course of the numerical simulation can, in principle, be described analytically. The solution of Eq. (1) can be given in the form

$$a(x,t) = \Big(\sum_{k=0}^{\infty} A_k(x)\psi_k(t)\Big)e^{-i\omega t}, \tag{4}$$

where $\{\psi_k(t)\}$ is the total system of orthogonal functions with period T.

An approximate solution of Eq. (1) can be written in the form of the finite sum

$$a_m(x,t) = \Big(\sum_{k=0}^{m} A_k(x)\psi_k(t)\Big)e^{-i\omega t}. \tag{5}$$

Since $a_m(x,t)$ is not an exact solution of Eq. (1), the difference

$$F(a_m) = \frac{\partial a_m}{\partial t} - a_m + (1+i\beta)|a_m|^2 a_m - (1+i\alpha)\frac{\partial^2 a_m}{\partial x^2} \tag{6}$$

cannot be identically equal to zero. Therefore, there arises the problem of minimization of this difference to a certain reasonable extent by a proper choice of $A_k(x)$.

We shall use the Galerkin method, which reduces the problem of finding $A_k(x)$ to the solution of a system of ordinary differential equations of the type

$$\int_0^T F\Big(\sum_{k=0}^{m} A_k(x)\psi_k(t)e^{-i\omega t}\Big)\bar{\psi}(t)e^{i\omega t}dt \quad (k = 0,\ldots,m), \tag{7}$$

where $\bar{\psi}$ is the complex conjugate of ψ. The system obtained is conservative and its phase volume is retained, i.e., its phase space contains no attractors. This explains the fact that to different initial conditions correspond different spatial distributions of the field in a steady-state regime.

To obtain an adequate description of the system behavior, we need a great number of harmonics. However, it can be assumed that for the solution of Eq. (1), which is periodic in time, there exists a Galerkin approximation of a sufficiently high order, which approximates this solution to a preset accuracy. That is, for each fixed $t = t_0$, the spatial distribution of the field $a(x,t_0)$ is approximated rather accurately by the solution of a finite-dimensional dynamic system (7) $a_m(x,t_0)$. We still have to clarify finally whether or not the exact solution for $a(x,t_0)$ is finite-dimensional.

This work was supported by the Russian Foundation for Fundamental Research (grant 94-02-03263-a).

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